A SIMPLE ANALYSIS OF THICK MICROSTRIP ON ANISOTROPIC SUBSTRATES

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Abstract

A simple variational numerical method is developed to yield efficient solutions to the problem of finite thickness microstrip lines on anisotropic and inhomogeneous substrates. The method relies on Galerkin's procedure with the expansion and testing functions being a set of exponential functions of the conductor perimeter variable.

I. Method of Solution

The existing methods to compute the characteristics of thick microstrip lines on anisotropic substrates rely mainly on the linearized approximation of the charge distribution on each side of the conductor [1], finite difference techniques [2] or variational methods [3].

In this note a simple technique is proposed which relies on the Galerkin method to resolve the problem of thick microstrip conductors on anisotropic substrates. The charge distribution on each side of the microstrip conductor is represented by a set of exponential functions in the form

$$q_m(s) = \sum_{i=1}^{N} C_m(k_i)\exp(k_i s) \tag{1}$$

where s is the local coordinate of the m'th side (m = 1,2,3,4) of the conductor, $C_m(k_i)$ represents unknown coefficients and the k_i are pivots to be determined. It can be shown that the charge distribution must satisfy for the n'th conductor the integral equation

$$V_n = \sum_{\substack{\ell,m \\ m\varepsilon C_\ell}} \oint_{C_\ell} ds' G(s,s') q_m(s') \tag{2}$$

where C_ℓ stands for the perimeter of the ℓ'th conductor and V_n is the voltage of the n'th conductor. The Green's function for the problem G(s,s'), corresponds to a quasistatic solution for a unit line of charge located at $-\infty < x' < \infty$, $H < y' < B$ (see figure 1) and it is given by [4], [5]

$$G(x,y;x',y') = \frac{1}{\pi\varepsilon_o}\int_{-\infty}^{\infty}\frac{dk}{k}\cos\left[\frac{k(x-x')}{n_x}\right]\sinh\left[\frac{k(B-y_>)}{n_x}\right].$$

$$\cdot\frac{n_x n_y \sinh\left[\frac{k(y_<-H)}{n_x}\right]\cosh\left(\frac{kH}{n_y}\right)+\sinh\left(\frac{kH}{n_y}\right)\cosh\left[\frac{k(y_<-H)}{n_x}\right]}{n_x n_y \sinh\left[\frac{k(B-H)}{n_x}\right]\cosh\left(\frac{kH}{n_y}\right)+\sinh\left(\frac{kH}{n_y}\right)\cosh\left[\frac{k(B-H)}{n_x}\right]} \tag{3}$$

where $y_< = \min(y,y')$, $y_> = \max(y,y')$, $n_x = (\varepsilon_x)^{1/2}$ and $n_y = (\varepsilon_y)^{1/2}$. The substrate is assumed to be anisotropic and it is characterized by a tensor permittivity with only its diagonal elements being nonzero. By employing a set of testing functions $\exp(k_i s)$ we formulate the appropriate inner products in equation (2) and we determine the coefficients $C_m(k_i)$. The total charge on the n'th conductor is of the form

$$Q_n = \sum_{m\varepsilon C_\ell} \oint_{C_\ell} ds' q_m(s') \tag{4}$$

We consider here the case of a single line of finite thickness and a convergence pattern of the method is shown in Table 1 for w/H = 1.0, B/H = 6.0, $\varepsilon_x = 9.4$, $\varepsilon_y = 11.6$ and t/H = 0.15. Obviously the choice of the k_i pivots effects the convergence pattern and it should be mentioned that these pivots have not been optimized. This has no limitation as to the value of the ratio t/H. In Table 2, the characteristic impedance Z_o and the ratio of the phase velocity v to the speed of light c are shown as functions of t/H. The case shown is for B/H = 6.0, w/H = 1 and for a homogeneous anisotropic substrate (single crystal sapphire i.e. $\varepsilon_x = 9.4$, $\varepsilon_y = 11.6$). Each result of Tables 1, 2 has been obtained in less than 6 sec CPU time in a CDC 6600 machine within 1% convergence error. This renders this method quite efficient for arbitrary t/H values.

Table 1. Convergence pattern for w/H = 1.0, B/H = 6, $\varepsilon_x = 9.4$, $\varepsilon_y = 11.6$ and t/H = 0.15

S_i Pivots		Capacitance per meter
Horizontal Sides	Vertical Sides	(Fd/m)
0.0,1.0	±0.17	0.1972×10^{-9}
0.0,1.0,2.0,3.0	±0.11,±0.33	0.1983×10^{-9}
0.0,0.6	±1.7	0.1990×10^{-9}
0.0,0.6,1.2,1.8	±1.1,±3.3	0.1973×10^{-9}

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Table 2. Dependence of Z_o and v to thickness t/H for w/H = 1 B/H = 6.0 and $\varepsilon_x = 9.4$, $\varepsilon_y = 11.6$.

t/H	$Z_o(\Omega)$	v/c
0.00	46.2	0.372
0.15	43.6	0.386
0.30	42.2	0.397
0.45	40.6	0.402

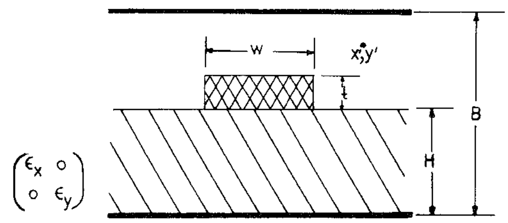

Figure 1. Geometry of Thick Microstrip.

II. Conclusions

A simple variational method based on expressing the unknown charge distribution in terms of a set of exponential function on each side of a microstrip conductor has been presented. This method is numerically quite efficient for the computation of the characteristics of microstrip lines of arbitrary thickness.

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